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MEASURING AND ASSESSING RISK

Abstract: The study and management of risk is the subject of various studies using a variety of methods, models, algorithms, programs, and systems of analysis and evaluation. The essence of risk can be defined broadly as a possible adverse deviation from the desired and expected outcome of the implementation of management decision or action due to the multiplicity and indeterminacy of the factors of the environment in which the action takes place, creating objective conditions for risk situations.

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Introduction

Risk is a category which is used in the natural sciences, mathematics, technical, social, medical and military sciences. Depending on the subject and the object of study, each of these sciences have specific methods, criteria and indicators for risk evaluation [3].

Measurement methods and risk assessment are divided into quantitative and qualitative. Most organizations prefer the quality, i. e. the expert assessment of the risks to quantitative (mathematical - statistics, modeling). This is due to the attitude and ability of risk managers, a lack of appropriate methods to quantify risks and the fact that not all risks can be assessed quantitatively, especially strategic and political ones.

In this context, the aim of the report is to analyze the methods of measurement and risk assessment.

Exposition

As we know, risk can be expressed mathematically as the probability of an event based on statistics and expert assessment.

Probabilistic risk measurement

The most common is *probabilistic risk measurement*, which can be determined by objective and subjective method, and by using special mathematical criteria for determining the degree of risk.

1) Objective method is manifested by classical, statistics, approximately probabilistic and indirect method:

-*Classic method* for determining the probability of an event. This method is based on equal opportunity and probability of all possible outcomes of a trial.Formula (1) is used to directly calculate the probability that the event A occurs.

$$P(\mathrm{A}) = \frac{m}{n}, \qquad (1)$$

where:

m – the number of cases favoring the occurrence of event A;

n – the total number of equally possible cases.

then

$$\sum_{i=1}^{n} P(A_i) = 1 \text{ и } 0 \leq P(A_i) \leq 1. \qquad (2)$$

-*Statistical method* for calculating the probability of an event. Calculate the relative frequency with which the event *A* occurs.

In many of the technical and economic areas we can not always use the classic definition of probabilities. In these cases, the following approach is used. If a *k* denotes the number of occurrence of an event *A*, when *n* independent trials have been carried out, the number *k* is called the absolute frequency of occurrence of the event *A* and the ratio $\frac{k}{n}$ - relative frequency. Therefore, the statistical probability can be defined as: the constant figure around which fluctuates the relative frequency $\frac{n}{k}$in a very large number of trials (3).

$$P(\mathrm{A})^* = \frac{k}{n}. \qquad (3)$$

-*Approximate probabilistic method.* When unable to obtain the sought probability distribution for all embodiments of the development of the event.

-*Indirect (qualitative method).* If the application of the above methods is impossible, other available indicators are measured, ones that indirectly characterize the risk.

2) Subjective method, where to measure risk subjective criteria are used, based on personal experience and reflections of the decision maker, expertise and consulting evaluations, etc.

3) Taking into account the uncertain factors whose law of distribution is unknown, special mathematical criteria are used for determining the degree of risk, such as: Wald criterion, Hurwicz criterion, criterion of Savage, criterion of Bayesian Laplace,criterion of extreme optimism, on the basis of which the decisions are taken [1, 3,4].

- The criterion of Wald (Q^w) is also known as *"the criterion of absolute pessimis"* (the strategy of the cautious loser). According to that criteria, for each alternative the minimum possible gains are set, and as an optimal one is accepted that for which profit so established is maximum, compared to others (4).

$$Q^w(A^*) = max\,min\,U_{jl}(B), \qquad (4)$$

where:

A^* – the optimal possible alternative;

$U_{jl}(B)$– value of the usefulness for the alternative.

Thus the decision maker can make the best choice expecting the worst scenario.

The choice made by the criterion of Wald can be explained by the following example. For each of the alternatives A_1, A_2 and A_3, of the table of usfulness (Table 1), is determined minimal state of nature φ_1, φ_2, or φ_3 (column min U_{jl} of Table 1). For optimal alternative is adopted A_3, as its value is the largest one (max min U_{jl} column of Table 1).

Table 1: *Table of usefulness of alternatives*

Alternative\ state of nature	φ_1	φ_2	φ_3	$min\ U_{jl}$	$\max min\ U_{jl}$
A_1	0	10	1	0	
A_2	2	6	4	2	
A_3	4	5	3	3	3*

- The criterion of Hurwicz (Q^H) is also called criterion of *pessimism-optimism*. According to the criterion, the true state of individual decision maker is either the worst or most favorable condition for it.

It is assumed that the probability for the person who is the decision-maker to be in unfavorable condition is $p^* = 1 - \alpha$, and the probability of occurrence of the most favorable condition is $p^{**} = 1 - p^* = \alpha$. With the coefficient α is assessed the degree of optimism. The bigger α, the bigger the optimism of the decision-maker, and vice versa.

According to the criterion of Hurwicz, for each alternative is determined the smallest and largest amount of usfulness, as optimum is determined that alternative for which the sum $[(1-\alpha).min\, U_{jl} + \alpha.max.U_{jl}]$ is the biggest (5):

$$Q^H(A^*) = max[(1-\alpha).min\, U_{jl} + \alpha.max.U_{jl}], \quad (5)$$

where:

A^* – the optimal alternative;

α – coefficient assessing the degree of optimism.

Table 2: *Values of usefulness for each alternative*

Alternative\ state of nature	φ_1	φ_2	φ_3	$min\, U_{jl}$	$\max U_{jl}$
A_1	0	10	1	0	10
A_2	2	6	4	2	6
A_3	4	5	3	3	5

The values of the coefficient α change from 0 to 1. When α = 0, the criterion of Hurwicz becomes the pessimistic criterion of Wald. When α = 1, the criterion can be recommended to persons favored by fortune. For the example of Table 2 can be determined the optimal solution.

The values of the criterion Q^H, calculated by formula (5) for different values of α (0 to 1), are given in Table 3.

With an asterisk in Table 3 are assigned the values of the criterion of Hurwicz which correspond to the optimal solution.

Table3:*Finding the optimal solution by the criterion of Hurwicz*

α	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
A_1	0	1	2	3	4*	5*	6*	7*	8*	9*	10*
A_2	2	2.4	2.8	3.2	3.6*	4	4.4	4.8	5.2	5.6	6
A_3	3*	3.2*	3.4*	3.6*	3.8	4	4.2	4.4	4.6	4.8	5

- The criterion of Laplace Q^L, also called *the criterion of apparent indifference,* is based on the already formulated principle of January Bernoulli of insufficient reason. According to him, if we don't have reasonable grounds to believe that a state of nature dominates over another, it is best to consider them equally possible (6).

$$p_1(\varphi_1) = p_2(\varphi_2) = p_3(\varphi_3) = \cdots = p_k(\varphi_k) = 1/k. \quad (6)$$

The assumption that the states of nature are equally possible destroys the uncertainty of the task and the choice of optimal alternative is performed as in the tasks with a risk. For the example presented, the decision upon this criterion is given in Table 4.

Table4:*Finding the optimal solution by the criterion of Laplace*

K=3 p=1/3	φ_1	φ_2	φ_3	Mj	Optimal solution
A_1	0	10	1	3.66	
A_2	2	6	4	4.00	4.00*
A_3	4	5	3	4.00	4.00*

- Criterion of *the maximum regret* of Savage Q^S. The rule for selection by the criterion of Savage says: The values of regret are determined r_{jl}, j = 1,2, ..., n and l = 1,2, ..., k, and for each alternative is defined the biggest regret max $max r_{jl}$. From the obtained values (column 5 of Table 6) is selected the smallest (7):

$$Q^S = min\, max r_{jl}. \qquad (7)$$

Table5:*Table of the usefulness of the alternatives*

Alternative \ state of nature	φ_1	φ_2	φ_3
A_1	0	10	1
A_2	2	6	4
A_3	4	5	3
Maximum profits for each state of nature U_{jl}	4	10	4

The criterion will be explained with the following example (Table 5). If the state of nature is φ_1, and the preferred alternative is A_1, the usefulness of the choice will be zero, as$u_{11} = 0$. In this case, the individual decision maker will regret for the unfortunately chosen solution. Only the choice of alternative A_3 in this state of nature (φ_1) will not be regrettable as it leads to the greatest possible profit$u_{31} = 4$. Similarly is determined the maximum regret for the other two states of nature- φ_2и φ_3.

Table 6:*Finding the optimal solution by the criterion of Savage*

K=3p=1/3	φ_1	φ_2	φ_3	$max r_{jl}$	Оптималнирешения
A_1	4	0	3	4	4*
A_2	2	4	0	4	4*
A_3	0	5	1	5	

The quantitative evaluation of regret is obtained as for each state of nature $\varphi_1, \varphi_2,\ and\ \varphi_3$ is determined the greatest possible usefulness $max U_{jl}$ and the differences $r_{jl} = max U_{jl} - U_{jl}$. The resulting estimated value r_{jl} evaluates the regret of the person who decides, if they made the choice Aj, and the state of nature was $\varphi_{1l} = 1,2, \ldots, k$.

For the example in Table 5 the values of regret $r_{jl}$and the optimal solution determined by the criterion of Savage are listed in Table 6.

There are also various modifications of the reviewed criteria on the basis of which to be defined optimal solutions.

Standard statistical features of risk measurement

These features allow for quantitative risk assessment and are presented by: mathematical expectation, dispersion, standard or average deviation, coefficient of variation and correlation coefficient[2, 6].

1) Mathematical expectation - $M(X)$.

Mathematical expectation is an important numerical characteristics, and with risky decisions before anything is valued the parameter " most expected result."Let's assume that the random value X can only accept meanings $x_1, x_2, \dots, x_n$, and their respective probabilities$p_1, p_2, \dots, p_n$, then the mathematical expectation M (X) of the random variable X is defined by the equality (8):

$$M(X) = x_1.p_1 + x_2.p_2 + \dots + x_n.p_n, \qquad (8)$$

where:

x_n - n-th possible outcome of the decision;

p_n- the probability of the n-th result;

n-the number possible results.

If the discrete random variable accepts a countable set of possible meanings, then the mathematical expectation

$$M(X) = \sum_{i=1}^{n} x_i.p_i \,, \qquad (9)$$

exists if the order in (9) is converging. Therefore, under the mathematical expectation of a discrete random variable, taking a finite number of values, is understood the sum of the products of all possible values of the random variable in their respective probabilities.

2) Dispersion and standard deviation.

Dispersion$D(X)$ of the discrete random variable X (10) is called the mathematical expectation of the square of the variance of the random variable X of its mathematical expectation$M(X)$.

$$D(X) = M[X - M(X)]^2. \qquad (10)$$

Besides the symbol D (X), the dispersion can also be referred to as$\sigma^2(X)$. For the calculation of the dispersion it is often more convenient to use the following formula (11):

$$\sigma^2(X) = D(X) = M(X^2) - [M(X)]^2, \qquad (11)$$

i. e.the dispersion equals the difference between the mathematical expectation of the square of the random variable X and the square of its mathematical expectation.

Standard deviation or medium-square deviation σ (X) of the random variable X is the square root of the dispersion and is calculated by the formula (12).

$$\sigma(x) = \sqrt{\sigma^2(x)} = \sqrt{D(X)}. \quad (12)$$

3) Coefficient of variation. The relative linear deviation is estimated using the coefficient of variation or volatility:

$$V(x) = \frac{\sigma(x)}{\bar{x}} . 100\%, \quad (13)$$

where:

$\bar{x} = \frac{\sum_{i=1}^{N} x_i}{N}$ is the arithmetic average value;

x_i – meanings of attributes (averaged values);

N – number of meanings of the attribute.

The coefficient of variation (13) is calculated as the ratio (12) of the average standard deviation to the arithmetic average attribute in percentage. The higher the coefficient of variation or volatility, the riskier is the solution.

In the specialized systems for risk assessment (mostly financial) are used *coherent measures of risk* [5] (*VAR - Value-at-Risk)*, giving information about current risk for the entire organization. Their risk can be measured by a single indicator that can be easily interpreted and compared with some base.

In calculating the indicator VAR are aggregated all the risks that would affect a particular organization based on certain assumptions. The analysis takes into account the specific requirements of the organization. The calculation of VAR is based on 3 parameters: time horizon that depends on the analyzed situation; confidence level - this is the level of tolerable risk (95-99%), and basic currency, in which is calculated the cost of risk.

Conclusion

For risk measurement most commonly is used the probability measurement and standard statistical characteristics. Probabilistic risk measurement is determined by an objective method, subjective method and using special mathematical criteria. Standard statistical characteristics are represented by: mathematical expectation, dispersion, standard or average deviation, coefficient of variation and correlation coefficient.

In close connection with the assessment - measurement and sources of risk, is risk management. We could say that the theory of probability and mathematical statistics are essential tools for risk management. This management includes the development of relevant standards, models, methods, software solutions, tools, etc. Moreover, the essence of risk management is in the decision making to maximize the many factors, conditions and circumstances that can be managed, and minimization of many factors, conditions and circumstances that cannot be managed, and within which the connection "cause - effect" is hidden or unknown.

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