



## TOPOGRAPHY ANALYSIS OF MERCURY USING DEM AND FRACTAL DIMENSION

**Abstract:** In the last decades, NASA's spacecraft missions of Mariner 10 and especially of MESSENGER have greatly increased our knowledge of the smallest planet within the solar system - Mercury. Based on their data, high resolution Digital Elevation Models (DEMs) for the planet's topography were created. This refined investigations and enabled researchers to use new and innovative scientific approaches. One of them is the fractal analysis. In the present study based on DEM and using „box-counting” method a fractal analysis of the Mercury's topography was performed. The study results show a global fractal dimension (FD) for the Mercury's topography of 1.530, FD –1.517 for the northern hemisphere and FD – 1.553 for the southern one.

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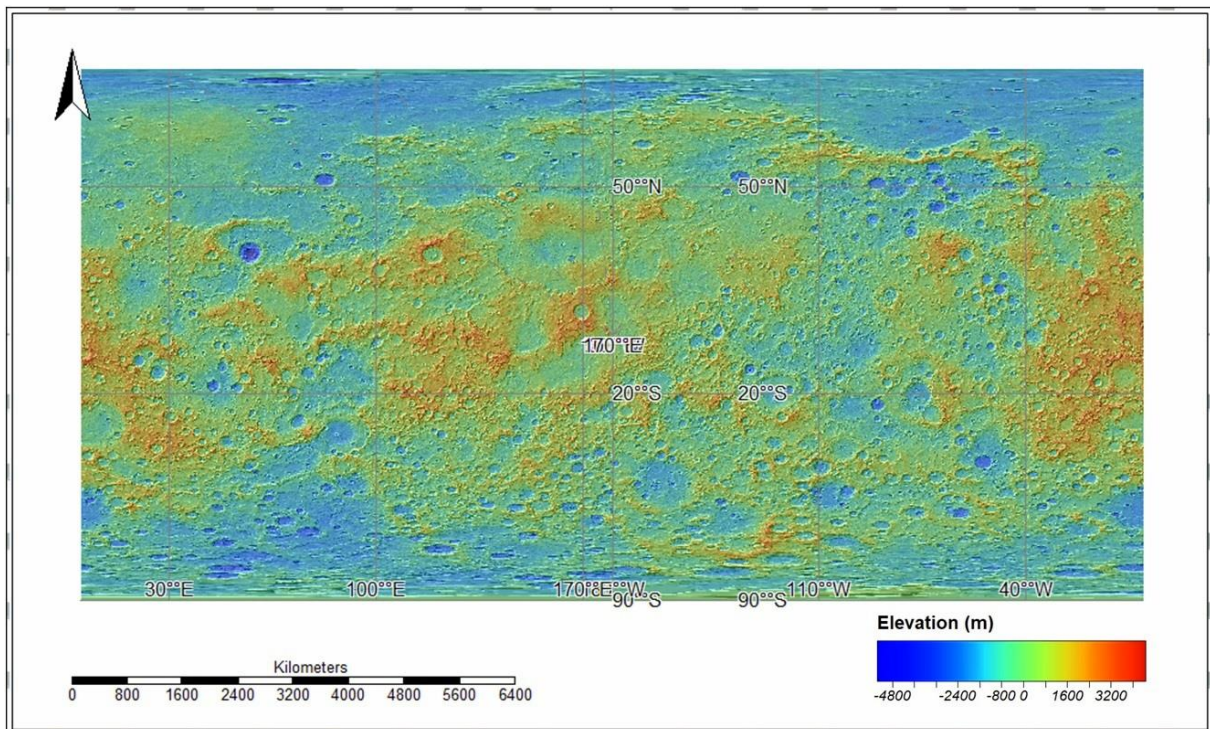
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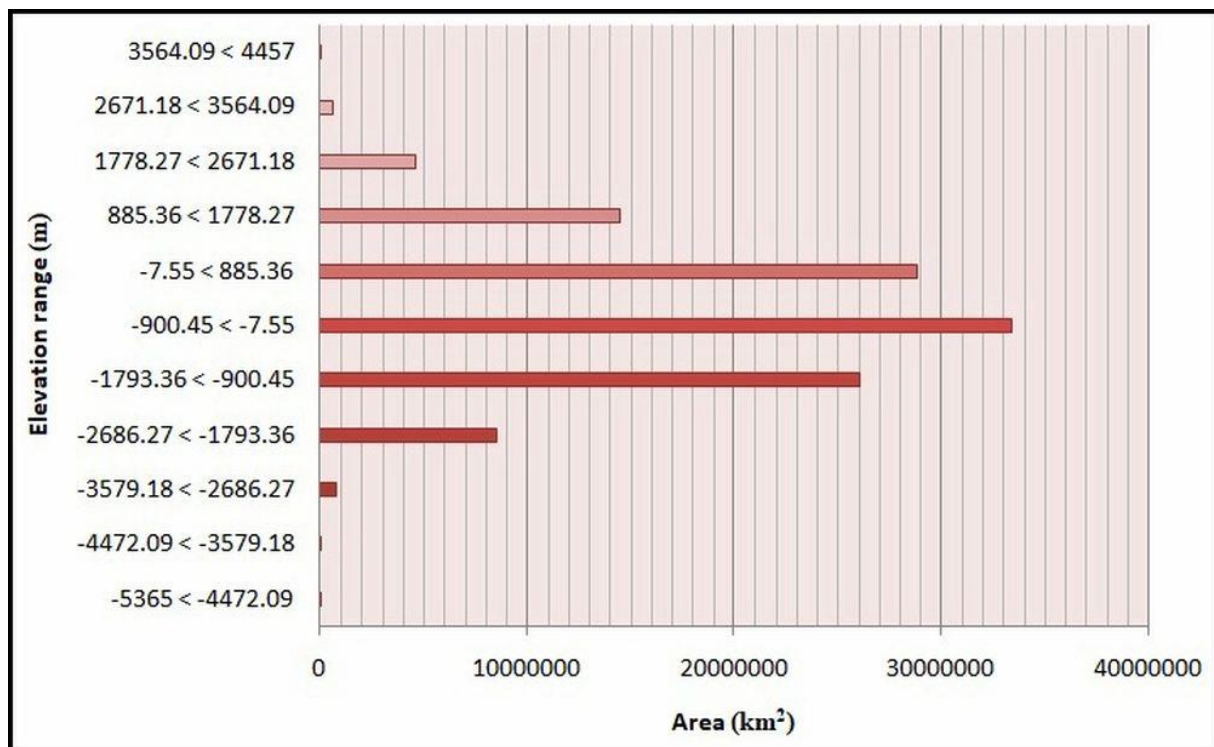
## INTRODUCTION

Prior to the Mariner 10 spacecraft mission [1974], the scientific community knew very little about the topographical features of the closest to the Sun planet – Mercury. In 1975 Trask and Guess created the first geologic map for the topography of Mercury. Within they generally recognized widespread heavily cratered plains and smooth plains, numerous younger craters with related deposits, hills and mountains (Fig.1). They found similarities in terms of the topography between Mercury and the Moon, which, in their view, is related to their similar geological evolution. In the following years based on data from MESSENGER missions [Solomon et al., 2001], our knowledge of the Mercury terrain peculiarities has been significantly enriched.

Mercury's terrain varies from -5365 m to 4457 m (Fig.2). Negative values occupy about 60%, while the positive ones occupy about 40% of the planet's surface [Ranguelov and Iliev, 2019b]. Within the northern hemisphere, the negative landforms prevail, while within the southern one the positive elevations are more widespread.



**Figure 1** Digital Elevation Model (DEM) of Mercury reveals main peculiarities of its terrain. The topography is dominated by impact craters and high hills. It is noteworthy that the topography of the polar regions of the planet is dominated by negative elevations, while the equatorial by positive ones.



**Figure 2** The graph shows that the negative values of the topography outweigh the positive ones. This is determined by the dominant role of the impact phenomena that shaped Mercury's appearance.

Over the last 3 decades, constantly emerging new data about solar system's planetary bodies have led to the emergence and development of a number of innovative methods of analysis and interpretation. An important place among them has the fractal analysis. Fractal geometry is a useful way to describe and characterize complex shapes and surfaces [Zhou and Lam, 2005]. Therefore, in the analysis of planets topography and physical properties, the fractal approach proves to be a very appropriate methodological tool. So far, the fractal paradigm has already been successfully used in the study of the geological and geophysical features of Mars [Turcotte, 1987; Demin et al., 2017], Venus [Turcotte, 1987; Demin et al., 2018] and our natural satellite - the Moon [Turcotte, 1987; Nefedjev, 2003; Baldassarri et al., 2008; Huang et al., 2009; Rosenburg et al, 2011; Cao et al., 2015; Kumar et al., 2016; Bray et al., 2018; Ranguelov et al., 2019]. Cabane et al. [1993] using fractals investigated the haze aerosols within Titan's atmosphere. Ranguelov and Iliev [2019a] revealed the fractal structure of the Solar system itself.

Concerning the planet Mercury, the use of fractal analysis has a very brief history. Only a few studies can be found in the scientific literature. Mancinelli et al. [2014] using fractals analyzed asteroid craters on the small planet. Ranguelov and Iliev [2019b] based on fractal dimension, explored the spatial variations of the terrain and gravity field of Mercury. In this regard, the insufficient number of publications on the topic determines one of the main tasks of the present study. For the purpose, based on the most up-to-date Digital Elevation Model (DEM) [Becker et al., 2016], Mercury's topography is divided into hypsometric belts over an interval of 500 m. In this way, using the fractal dimension approach, the patterns in the spatial distribution of the Mercury terrain are fully revealed.

## METHODS AND DATA

### Box-counting method for fractal dimension (FD) calculation

There are numerous methods proposed to calculate the fractal dimension [FD] of an image [Zhou and Lam, 2005]. In this study, we calculate FD for Mercury's topography using well known „box-counting” method [Mandelbrot, 1982; Voss, 1986; Hirata, 1989; Sarkar and Chaudhuri, 1992]. The analytical procedures involve several steps. First, Mercury's topography is divided into individual raster images using 500-meters interval. Then, each raster image is covered with series of boxes and software calculated the total number of boxes containing any information. In this way, the FD is determined based on the famous formula (Mandelbrot, 1977):

$$D = \lim_{r \rightarrow 0} \frac{\log(N(r))}{\log(1/r)} \quad (1)$$

where  $N(r)$  is the number of boxes that contain at least one piece of spatial data, and  $r$  is the boxes length (size).

Finally, a fractal dimension is calculated for each hypsometric interval. In this way it is possible to analyze the degree of spatial fragmentation of Mercury's terrain - by 500- meters hypsometric belts, for the planet as a whole and for each of two hemispheres (for comparison purposes).

### Data and software

The analysis of the Mercury's topography is performed using digital elevation data from Mercury Global Digital Elevation Model (DEM) v2 [Becker et al., 2016] derived from MESSENGER spacecraft missions [Solomon et al., 2001]. The DEM is created at 665x665 m pixel size.

The digital terrain analysis has been performed using Geographic Information System (GIS) – SAGA-GIS [Conrad et al., 2015] and ImageJ software.

## RESULTS AND DISCUSSION

Fractal analysis of the topography of Mercury shows a fractal dimension (FD) of 1.530 (Table 1). This means moderate values of terrain fragmentation as a whole. In general, negative values (FD – 1.763) have a more non-linear spatial distribution than positive ones (FD – 1.573). This is determined by the turbulent geological history of Mercury and the high frequency of impact phenomena. The highest values of spatial fragmentation have the hypsometric belts – -500 – 0 m (FD – 1.792), -1000 – -500 m (FD – 1.754) and 0 – 500 m (FD – 1.747). The results show that moderately deep impact craters and plain – slightly hilly terrains have the most non-linear or chaotic spatial distribution.

**Table 1** Fractal analysis of the Mercury's topography

Positive elevations range (m)	Area (km <sup>2</sup> )	FD	Negative elevations range (m)	Area (km <sup>2</sup> )	FD	Overall (m)	FD
0 – 500	18346908	1.747	-5365 – -5000	7283	1.676	-5365 – 4457	
500 – 1000	12703115	1.645	-5000 – -4500	8692	1.677		
1000 – 1500	8705428	1.541	-4500 – -4000	11746	1.660		
1500 – 2000	5096187	1.473	-4000 – -3500	88566	1.608		
2000 – 2500	2379888	1.419	-3500 – -3000	101054	1.477		
2500 – 3000	791690	1.425	-3000 – -2500	865339	1.376		
3000 – 3500	186176	1.509	-2500 – -2000	4380376	1.377		
3500 – 4000	31832	1.567	-2000 – -1500	10605781	1.500		
4000 – 4457	2466	1.650	-1500 – -1000	15603946	1.641		
			-1000 – -500	17809404	1.754		
			-500 – 0	19435777	1.792		
<b>Mean</b>		<b>1.572</b>	<b>Mean</b>		<b>1.763</b>	<b>Mean</b>	<b>1.530</b>

The fractal analysis of the two hemispheres of the planet shows that the southern hemisphere of Mercury (Table 3) has a slightly higher fractal dimension than the northern one (Table 2 and 4). The FD ratio is 1.553 : 1.517. Regarding the elevation type of terrain, in both hemispheres the non-linear distribution has negative elevation values, but within the northern hemisphere the difference is more pronounced (FD – 1.500 for the positive values and FD – 1.687 for the negative ones). In the southern hemisphere, this ratio is 1.563 : 1.684 (Table 4).

**Table 2** Fractal analysis of the Mercury's Northern Hemisphere topography

Positive Elevations range (in meters)	FD	Negative Elevations range (in meters)	FD	Overall (in meters)	FD
0 – 500	1.727	-5365 – -5000	0.418	-5365 – 4308	
500 – 1000	1.572	-5000 – -4500	0.697		
1000 – 1500	1.413	-4500 – -4000	0.651		
1500 – 2000	1.231	-4000 – -3500	0.703		
2000 – 2500	0.963	-3500 – -3000	0.684		
2500 – 3000	0.796	-3000 – -2500	0.717		
3000 – 3500	0.585	-2500 – -2000	1.095		
3500 – 4000	0.414	-2000 – -1500	1.392		
4000 – 4308	n/a	-1500 – -1000	1.575		
		-1000 – -500	1.702		
		-500 – 0	1.754		
<b>Mean</b>	<b>1.500</b>	<b>Mean</b>	<b>1.687</b>	<b>Mean</b>	<b>1.517</b>

**Table 3** Fractal analysis of the Mercury's Southern Hemisphere topography

Positive Elevations range (in meters)	FD	Negative Elevations range (in meters)	FD	Overall (in meters)	FD
0 – 500	1.698	-4612 – -4000	0.971	-4612 – 4457	
500 – 1000	1.581	-4000 – -3500	0.947		



1000 – 1500	1.442	-3500 – -3000	0.874		
1500 – 2000	1.315	-3000 – -2500	0.925		
2000 – 2500	1.179	-2500 – -2000	1.076		
2500 – 3000	1.090	-2000 – -1500	1.302		
3000 – 3500	1.007	-1500 – -1000	1.523		
3500 – 4000	1.012	-1000 – -500	1.670		
4000 – 4457	1.013	-500 – 0	1.770		
<b>Mean</b>	<b>1.563</b>	<b>Mean</b>	<b>1.684</b>	<b>Mean</b>	<b>1.553</b>

**Table 4** Statistical summary of the fractal dimensions (FDs) spread within the Mercury's hemispheres. The results plotted on the table confirm the more non-linear distribution of the 500-meters hypsometric belts within the northern hemisphere of the small planet. However, within the southern hemisphere, there are differences in the positive and negative values, while within the northern one the values are similar.

	<b>FDmin</b>	<b>FDmax</b>	<b>FDmean</b>	<b>FDStDev</b>
<b>Northern hemisphere (overall)</b>	0.414	1.754	1.057	0.468
<i>Positive values</i>	0.414	1.727	1.087	0.474
<i>Negative values</i>	0.418	1.754	1.035	0.485
<b>Southern hemisphere (overall)</b>	0.874	1.770	1.244	0.299
<i>Positive values</i>	1.007	1.698	1.259	0.262
<i>Negative values</i>	0.874	1.770	1.228	0.347

Within both hemispheres, the tendency for most non-linear spatial distribution of the hypsometric belts -500 – 0 m (FD – 1.754 for the northern hemispheres and FD – 1.770 for the southern one), -1000 – -500 m (FD – 1.702 : 1.670) and 0 – 500 m (FD – 1.727 : 1.698) is preserving. However, within the northern hemisphere, the absolute maximum elevation values have a lower FD than negative ones. Within the southern hemisphere, the opposite trend is observed, the absolute maximum values have a higher FD than absolute negative ones. Ignoring the apparent differences between the two hemispheres, it is evident that, as a whole, the Mercury's terrain is characterized by a balanced distribution of the 500-meters hypsometric belts (Table 4). This interesting feature requires more in-depth research in the future.

## CONCLUSION

In the course of the presented study, a digital analysis of the topography of Mercury was performed. For this purpose, a combined approach – DEM and fractal dimension – was used. The obtained results showed regularity in the spatial distribution of individual 500- meters hypsometric belts within the planet as a whole. On the other hand, there are some differences between the two hemispheres of the planet in terms of different distribution of the absolute elevation values. Some peculiarities were revealed. On the one hand the Northern hemisphere keeps low level of fractal dimensions for negative elevation ranges (FD 0.418 and 0.717 between -5365m and – 3000m and then a jump to (FD1.075 – 1.754) between -3000m and 0m. For the positive elevation ranges the respective values are FD 0.414 and 0.963 for 2000m – 4300m and the jump over FD1.000 is FD1.231and 1.727 for 0m to +2000m.

On the other hand for the Southern hemisphere similar values FD are greater than 1.000 for all positive ranges and for the negative ones again there is a jump for FD from approximately 0.9 to greater than 1.000 at the level of the negative elevation range -2500m and -2000m. These peculiarities mean that the levels around 0+/- 2000-3000m in both hemispheres have larger fractality than the extreme variations in positive and negative DEM ranges.

This distribution reflects the varying intensity of the internal generic geological processes and external impacts that created and shaped the surface of the small planet.

These peculiarities rise the fundamental question - is this behavior due to the genesis of the high and low lands of Mercury or they are a product of evolution of the planet. Or maybe by both?

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